Modeling the behaviour of Economic Agents as a Response to Information on Tax Audits

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Table of contents:

The Related Models of Tax Control

- Approaches:
  - decision theory;
  - contract theory;
  - game theory;
  - probabilistic modeling.
- Hierarchical Structure: «principal-agent»
- Risk-neutral agents;
- The forms of solutions:
  - agent's optimal reporting rule [Reinganum Wilde, 1984]
  - optimal contract [Sanchez, Sobel, 1993], [Chander, Wilde, 1998]
  - threshold rule [Vasin et al., 1993 – 2002]

The Related Models of Tax Control

From ([Bure, Kumacheva, 2010]): Risk-neutral taxpayers refuse their evasion from high level of income H to low level of income L if

$$P_L \geq P^*,$$

where

- P_L is probability of audit of agents, who declared low level of income;
- P^* is the optimal value of auditing probability.

The Idea: dissemination of information about tax auditing

Problem 1: Limited budget of the tax authority

$\Downarrow$

reaching optimal values of auditing probabilities is extremely rare

$\Downarrow$

the tax authority needs to find additional ways to stimulate taxpayers to pay taxes

Problem 2: Full information in the game-theoretical models

$\Downarrow$

The dissemination of information about increased probability of tax auditing.

The Idea: dissemination of information about tax auditing

The approaches of studying of the information spreading in the models of tax control:

- epidemic processes (SIR and SIS models) — [Gubar, Kumacheva, Zhitkova, Porokhnyavaya, 2015, 2016];
- evolutionary processes — [Gubar, Kumacheva, Zhitkova, Kurnosyh, Skovorodina 2017];
- random processes (Markov processes) — [Gubar, Kumacheva, Zhitkova, Tomilina, 2018].

Models of Tax Control under Information Diffusion:

1. **Model 1**: risk-neutral agents [Gubar, Kumacheva, Zhitkova, Tomilina, 2019 (GTM – ISDG)];
2. **Model 2**: agents with different risk-propensity: risk-averse, risk-neutral and risk-loving.

Model 1: Risk-neutral Agents

Based on the Static Model of Tax Control [Bure, Kumacheva, 2010]:

- a population of n taxpayers;
- ξ – true income, η – declared income; $\eta \leq \xi$;
- $\xi, \eta \in \{L, H\}$, where $0 < L < H$;
- two groups of population n_H and n_L: $n_L + n_H = n$;
- P_L – probability of audit of agents, who declared $\eta = L$;
- the tax evader should pay $(\theta + \pi)(\xi - \eta)$ constants θ and π are tax and penalty rates correspondingly;
- c – the unit cost of auditing;
- the optimal value of auditing probability:

$$P^* = \frac{\theta}{\theta + \pi}.$$

The Static Model of Tax Control: Taxpayers' profit functions

$$u\left(L(L)\right) = (1-\theta)\cdot L; \tag{1}$$

$$u\left(H(H)\right) = (1-\theta)\cdot H; \tag{2}$$

$$u\left(L(H)\right) = H - \theta L - P_L(\theta+\pi)(H-L). \tag{3}$$

Dissemination of information about tax auditing: Assumptions

- The considered population **is restricted** by the subpopulation of taxpayers with high level H of income: n_H;
- if **no** information: the total population evades ($n_H = n_{ev}$, where n_{ev} – the number of evaders);
- if **information injects** in the initial time moment: **the number** $n^0_{inf} = n^0_{nev}$ **of informed** about increased probability of tax auditing and $\Longrightarrow$ decided **not to evade**; ν^0_{inf} – the value (fraction) of the **informational injection** at the initial time moment ($\nu^0_{inf} = \nu_{inf}(t_0)$);
- in each time moment: $n_H = n_{ev}(t) + n_{nev}(t)$ (or $\nu_{nev}(t) + \nu_{ev}(t) = 1$), $t \in [0, T]$.
- ν^T_{nev} – the share of those who don't evade at the moment $t = T$, ν^T_{ev} – the share of those who continue to evade taxation at the moment $t = T$;
- c_{inf} – the unit cost of such injection ($c_{inf} << c$).

Total Tax Revenue in the **Model 1**: Initial and Final State

TTR_0^N – the total tax revenue in the absence of information, when only agents with true income level L pay:

$$TTR_0^N = n_L \theta L + n_H \left(\theta L + P_L (\theta + \pi)(H - L)\right) - n P_L c, \tag{4}$$

TTR_T^N – the total tax revenue when the information was spread and the system has come to a steady state (time moment T):

$$\begin{aligned} TTR_T^N = n_L \theta L + n_H \quad & \left(\nu_{nev}^T \theta H + \nu_{ev}^T (\theta L + P_L (\theta + \pi)(H - L))\right) - \\ & - n (P_L c + \nu_{inf}^0 c_{inf}), \end{aligned} \tag{5}$$

Model 2: Agents with Different Risk-propensity

Three subgroups of the total population of taxpayers $(\nu_a + \nu_n + \nu_l = 1)$

- risk-averse (share is ν_a);
- risk-neutral (share is ν_n);
- risk-loving (share is ν_l)

differ from each other by the various agents' behaviour profiles in the similar external conditions $\Rightarrow$ their response on the same information should be different.

Total Tax Revenue in the **Model 2**: Initial and Final State

TTR_0^R – the total tax revenue in the absence of information, when only taxpayers with low level of income L and risk avoiding agents pay:

$$TTR_0^R = n_L\theta L + n_H\left(\nu_a\theta H + (1-\nu_a)P_L(\theta+\pi)(H-L)\right) - nP_L\,c. \qquad (6)$$

TTR_T^R – the total tax revenue when the information was spread and the system has come to a steady state (time moment T):

$$\begin{aligned} TTR_T^R = n_L\theta L \quad & +n_H\left(\nu_a\theta H + (1-\nu_a)(\nu_{nev}^T\theta H + \nu_{ev}^T(\theta L + P_L(\theta+\pi)(H-L)))\right) \\ & -n(P_L\,c + \nu_{inf}^0 c_{inf}). \end{aligned} \qquad (7)$$

Evolutionary Network Model: Assumptions

- The social connection of each taxpayer can be represented as networks of various modifications;
- from [Riehl, Cao, 2015], [Gubar, Kumacheva, Zhitkova, Kurnosykh, 2017]: the process of disseminating information about inspections in the taxpayers' network is **similar to the evolutionary process in the population of economic agents**;
- the algorithm based on the proportional imitation rule [Weibull, 1995], [Sandholm, 2010] is used.

Evolutionary Process: the Algorithm Based on the Proportional Imitation Rule

- There is a well-mixed population of economic agents (taxpayers);
- the instant communications between taxpayers can be defined by two-players symmetric bimatrix game $\Gamma(A, B)$;
- every taxpayer can choose one of two strategies $X = \{ev, nev\}$, where ev – to evade, nev – not evade;
- for the cases, when taxpayers of different types meet each other, matrix of payoffs can be written in one of the following form.

Evolutionary Model: the Types of Payoff Matrixes

- The Prisoner's Dilemma:
  - to cooperate – to pay taxes;
  - to defeat – to evade;
- The Stag Hunt game:
  - collective strategy (to hunt a stag) – to pay taxes;
  - individual strategy (to hunt a hare) – to evade;
- The Hawk-Dove game:
  - passive behavior (to be a Dove) – to pay taxes;
  - aggressive behavior (to be a Hawk) – to evade.

Evolutionary Network Model: The Ideas and Structure

- Consider indirect network $G = (N, K)$, where
  - $N = \{1, \dots, n_H\}$ is a set of economic agents;
  - $K \subset N \times N$ is an edge set (each edge in K represents two-player symmetric game between connected taxpayers).
- The taxpayers choose strategies from a binary set $X = \{ev, nev\}$ and receive payoffs according to the matrix of payoffs.
- At each instant time moment agents use a single strategy against all opponents and thus the games occurs simultaneously.
- The strategy state: $x(t) = (x_1(t), \dots, x_{n_H}(t))^T$, $x_i(t) \in X$, where $x_i(t) \in X$ is a strategy of taxpayer i, $i = \overline{1, n_H}$, at time moment t.

The Application: The Evolutionary Model with Network Structure

Aggregated payoff of agent i will be defined as in [Riehl and Cao, 2015]:

$$u_i = \omega_i \sum_{j \in M_i} a_{x_i(t),x_j(t)}, \tag{8}$$

where

- $a_{x_i(t),x_j(t)}$ is a component of payoff matrix A;
- $M_i := \{j \in K : \{i,j\} \in K\}$ is a set of neighbors for taxpayer i;
- weighted coefficient $\omega_i = 1$ for cumulative payoffs and $\omega_i = \frac{1}{|M_i|}$ for averaged payoffs.

Vector of payoffs of the total population is

$$u(t) = (u_1(t), \ldots, u_{n_H}(t))^T.$$

Evolutionary Network model: The Algorithm Based on the Proportional Imitation Rule

The state of population will be changed according to the **Proportional Imitation Rule** (see [Weibull, 1995], [Sandholm, 2010]):

Each agent chooses a neighbor randomly and if this neighbor received a higher payoff by using a different strategy, then the agent will switch with a probability proportional to the payoff difference:

$$p\left(x_i(t+1) = x_j(t)\right) := \left[\frac{\lambda}{|M_i|}(u_j(t) - u_i(t))\right]_0^1 \qquad (9)$$

for each agent $i \in K$, where $j \in M_i$ is a uniformly randomly chosen neighbor, $\lambda > 0$ is an arbitrary rate constant, and the notation $[z]_0^1$ indicates $\max(0, \min(1, z))$.

Modifications of Imitation Rules

- **Rule 1.** *Random neighbor.* When a taxpayer i receives an opportunity to revise her strategy then she chooses an exampled agent at random with equal probability to all connected neighbors.
- **Rule 2.** *Neighbor with the highest payoff.* When agent i receives an opportunity to revise her strategy then she considers current payoffs of all taxpayers and chooses an agent (or a set of agents) with maximum payoff. If there are several agents with maximum payoff then an exampled agent is chosen at random between them.
- **Rule 3.** *The most influenceable neighbor.* Firstly, taxpayer i estimates a number of connections of all nearest neighbors and selects an exampled agent from the set of agents with the maximum links. If there are several agents with maximum links, then an opponent is selected at each iteration of the dynamic process at random in the subset of influenceable agents.

Numerical Simulation: Visualisation of the Network

- the number of nodes (the size of total population): $N = n_H = 25$;
- initial distribution of evaders and honest taxpayers is the same: $n^0_{ev} = 13$, $n^0_{nev} = 12$ and two levels of income: $L = 12500$ and $H = 50000$;
- $TTR_0 = 52082.86$ rub. for each case, see (6);
- agents who use strategy:
  - "to pay taxes" (*nev*) – squares □;
  - "to evade" (*ev*) – circles ◯;
- the risk status of agents – the colors in the figures:
  - risk-averse taxpayers – green;
  - risk-neutrals – red;
  - risk-loving – blue;
- modification of the network:
  - strongly connected network (probability of link: 1/3);
  - weakly connected network (probability of link: 1/10);
  - grid.

Numerical simulation: Example 1

- configuration of the network: grid;
- instant game: Prisoner's Dilemma;
- imitation rule: rule 3 (the most influenceable neighbor);
- method of computing of agent's profit: cumulative;

Numerical simulation: Example 1

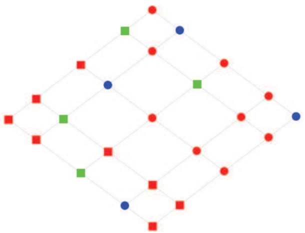

Figure: Experiment I. Initial state of the population is $n^0_{ev} = 13$, $n^0_{nev} = 12$, initial $TTR^R_0 = 52082.86$;

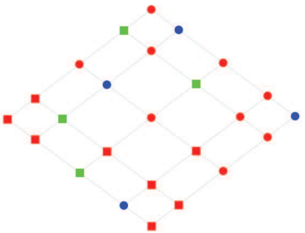

Figure: Experiment I. Final state of the population is $n^T_{ev} = 15$, $n^T_{nev} = 10$, final $TTR^R_T = 65210.50$.

$TTR^R_T = 65210.50$ rub. $\Longrightarrow$ total tax revenue increases.

The Dynamics of TTR^R Depending on the Number of Honest Taxpayers

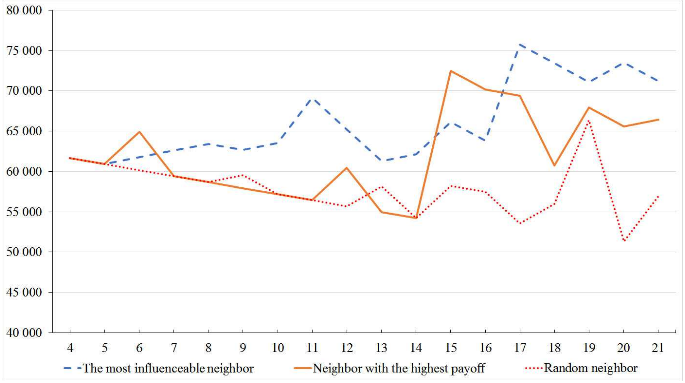

Figure: Experiment Series I. The axis: ordinate – the values of TTR^R, abscissa – the number of honest taxpayers.

Prisoner's Dilemma: Different Imitation Rules

The maximum values of TTR_T^R obtained by using different imitation rules for the corresponding initial and final data are following:

- The most influenceable neighbor: $TTR_T^R = 75758.7$, initial state of the population is $n_{ev}^0 = 8, n_{nev}^0 = 17$; final state of population is $n_{ev}^T = 6, n_{nev}^T = 19$;
- Neighbor with the highest payoff: $TTR_T^R = 72490.78$, initial state of the population is $n_{ev}^0 = 10, n_{nev}^0 = 15$; final state of population is $n_{ev}^T = 9, n_{nev}^T = 16$;
- Random neighbor: $TTR_T^R = 66336.52$, initial state of the population is $n_{ev}^0 = 6, n_{nev}^0 = 19$; final state of population is $n_{ev}^T = 11, n_{nev}^T = 14$.

Prisoner's Dilemma: Results

Dynamics of the total tax revenue demonstrate different behavior for different imitation rules:

- "the most influenceable neighbor" and "neighbor with the highest payoff" increase the total revenue with the increasing of the share of honest taxpayers;
- "random neighbor" decrease the value of TTR^R .

Numerical simulation: Example 2

- configuration of the network: strongly connected network;
- instant game: Stag Hunt;
- imitation rule: rule 2 (the neighbor with highest payoff);
- method of computing of agent's profit: cumulative.

Numerical simulation: Example 2

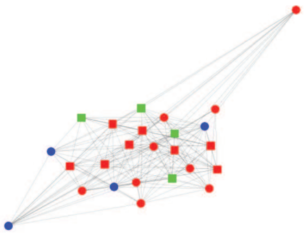

Figure: Experiment II. Initial state of the population is $n^0_{ev} = 13$, $n^0_{nev} = 12$, initial $TTR^R_0 = 52082.86$;

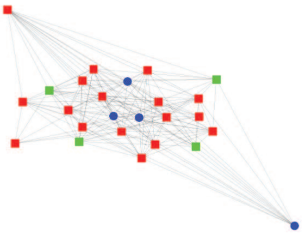

Figure: Experiment II. Final state of the population is $n^T_{ev} = 4$, $n^T_{nev} = 21$, final $TTR^R_T = 82657.93$.

$TTR^R_T = 82657.93$ rub. $\Longrightarrow$ total tax revenue increases.

The Dynamics of TTR^R Depending on the Number of Honest Taxpayers

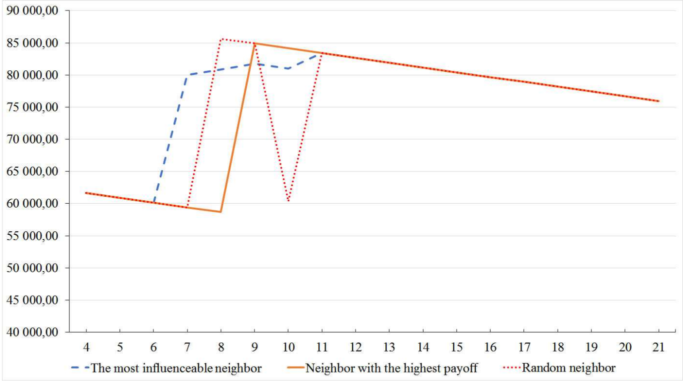

Figure: Experiment Series II. The axis: ordinate – the values of TTR^R, abscissa – the number of honest taxpayers.

Stag Hunt: Different Imitation Rules

The maximum values of TTR_T^R obtained by using different imitation rules for the corresponding initial and final data are following:

- The most influenceable neighbor: $TTR_T^R = 83403.43$, initial state of the population is $n_{ev}^0 = 14, n_{nev}^0 = 11$; final state of population is $n_{ev}^T = 4, n_{nev}^T = 21$;
- Neighbor with the highest payoff: $TTR_T^R = 84894.43$, initial state of the population is $n_{ev}^0 = 16, n_{nev}^0 = 9$; final state of population is $n_{ev}^T = 4, n_{nev}^T = 21$;
- Random neighbor: $TTR_T^R = 85639.93$, initial state of the population is $n_{ev}^0 = 17, n_{nev}^0 = 8$; final state of population is $n_{ev}^T = 4, n_{nev}^T = 21$.

Stag Hunt: Results

Dynamics of the total revenue demonstrate different behavior for different imitation rules:

- from the initial number of non-evaders to $n_{nev} = 7, \ldots, 9$ the value of total tax revenue archives its peaks and then decreases;
- however the general trend is the increasing of total tax revenue for all imitation rules.

Numerical simulation: Example 3

- configuration of the network: weakly connected network;
- instant game: Hawk Dove;
- imitation rule: rule 1 (random neighbor);
- method of computing of agent's profit: cumulative.

Numerical simulation: Example 3

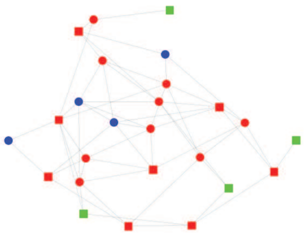

Figure: Experiment III. Initial state of the population is $n^0_{ev} = 13$, $n^0_{nev} = 12$, initial $TTR^R_0 = 52082.86$;

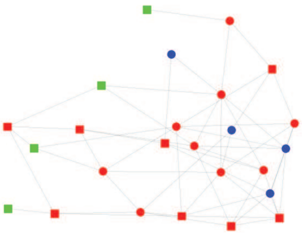

Figure: Experiment III. Final state of the population is $n^T_{ev} = 7$, $n^T_{nev} = 18$, final $TTR^R_T = 77899.54$.

$TTR^R_T = 77899.54$ rub. $\Longrightarrow$ total tax revenue increases.

The Dynamics of TTR^R Depending on the Number of Honest Taxpayers

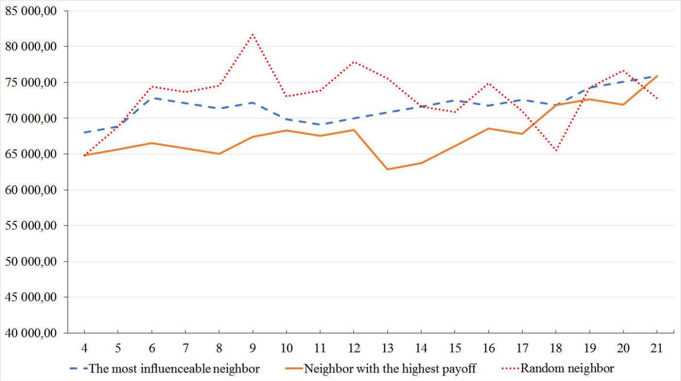

Figure: Experiment Series III. The ordinate axis represents the values of TTR^R, and the abscissa axis shows the number of honest taxpayers.

Hawk Dove: Different Imitation Rules

The maximum values of TTR_T^R obtained by using different imitation rules for the corresponding initial and final data are following:

- The most influenceable neighbor: $TTR_T^R = 75948.43$, initial state of the population is $n_{ev}^0 = 4, n_{nev}^0 = 21$; final state of population is $n_{ev}^T = 4, n_{nev}^T = 21$;
- Neighbor with the highest payoff: $TTR_T^R = 75948.43$, initial state of the population is $n_{ev}^0 = 4, n_{nev}^0 = 21$; final state of population is $n_{ev}^T = 4, n_{nev}^T = 21$;
- Random neighbor: $TTR_T^R = 81722.17$, initial state of the population is $n_{ev}^0 = 16, n_{nev}^0 = 9$; final state of population is $n_{ev}^T = 6, n_{nev}^T = 19$.

Result: all dynamics fluctuate around mean value with weak tendency of increasing total revenue.

Results: The Model

- tax control;
- different risk statuses of agents of the population;
- evolutionary process of information spreading is based on:
  - instant bimatrix games: “Prisoner’s Dilemma”, “Stag Hunt”, “Hawks and Doves”;
  - modifications of imitation rules: “Random neighbor”, “Neighbor with the highest payoff”, “The most influenceable neighbor”;
- network of different topology: grid, strongly connected and weakly connected random graphs.

Results: Experiment

- 18 different cases of the initial distribution of risk statuses in the taxpayer population;
- for each initial distribution corresponding simulation was repeated 10^2 times to obtain statistically significant data;
- each experiments was performed for two types of payoffs: cumulative and average.

Results and Conclusions: Instant Games

- Prisoner's Dilemma: the strategy of evasion is stable (chosen by the majority of taxpayers in the most number of experiments).
- Hawks and Doves: for any combination of imitations protocols, graphs and payoffs, agents prefer strategy "to pay".
- Stag Hunt: generally similar; the case of a weakly connected graph — a tendency of the equiprobable choice of both strategies — using "The Most Influential Neighbor" and "Neighbor with the highest payoff"; the largest difference in the choice of strategies — using "The Random Neighbor protocol".

Results and Conclusions

- The structure of the bimatrix game has the most influence on the final distribution of taxpayers.
- Two trends in changes of the total tax revenue of the system:
  1. if the number of honest taxpayers prevails over the number of evaders in the final distribution then a value of TTR_T^R increases in comparison with the initial value of TTR_R^0;
  2. in the rare cases TTR_R grows even if a number of evaders is larger then a number of honest taxpayers in final distribution (the collection of taxes and penalties increases) — the probability of such event is small.

General Conclusions

- The propagation information about possible tax audit brings a positive effect for the total revenue of fiscal system and increases total amount of honest taxpayers: the reduction of risk-loving agents helps to provide one of the fundamental principles of a tax system – the principle of fair taxation.

- Knowledge of the structure of payoff matrix and modification of imitation rules simplifies the scenario analysis of the impact of information on the effectiveness of tax control.

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Thank You for Your Attention!

Additional Information

Evolutionary Model: the Prisoner's Dilemma

Payoff matrix A, $B = A^T$:

	C	D
C	$(\overline{u} + SW), (\overline{u} + SW)$	$(\overline{u} - SW, u(L(H)))$
D	$(u(L(H)), \overline{u} - SW)$	$(\overline{u}, \overline{u})$

where

- C – to cooperate (to pay taxes), D – to defeat (to evade);
- $\overline{u} = 1/2u(L(L)) + 1/2u(H(H))$ – average profit of the "mean" agent;
- SW – social welfare, obtained for the participation in social consolidation.

Evolutionary Model: the Stag Hunt game

Payoff matrix A, $B = A^T$:

	S	I
S	$(\overline{u} + SW, \overline{u} + SW)$	$(0, \overline{u} - SW)$
I	$(\overline{u} - SW, 0)$	$(\overline{u} - SW, \overline{u} - SW)$

- where S – to hunt a stag (to pay taxes), I – individual strategy "to hunt a hare" (to evade).

Evolutionary Model: the Hawk-Dove game

Payoff matrix A, $B = A^T$:

	F	D
F	$\left(\frac{u(L(H)) - EL}{2}, \frac{u(L(H)) - EL}{2}\right)$	$(\overline{u} + SW, 0)$
D	$(0, \overline{u} + SW)$	$(\frac{\overline{u}+SW}{2}, \frac{\overline{u}+SW}{2})$

where

- F – to be a Hawk (aggressive behavior, i.e. to evade), D – to be a Dove (passive behavior, i.e. to pay taxes);
- EL is the level of evasion: $EL = (\theta + \pi)(H - L)$;
- $u(L(H)) << EL$: it works for the large values of parameters θ and π or when there is a big difference $(H - L)$.

Numerical simulation: parameters

- tax rate: $\theta = 13\%$;
- penalty rate: $\pi = 13\%$;
- optimal value of the probability of audit: $P^* = 0.5$;
- actual value of the probabilities of audit: $P_L = 0.1$;
- shares: $\nu_a = 17\%$ – risk-avoidants (non-evaders in the initial time moment), 65% – risk-neutral agents, 18% – risk-loving agents;
- unit cost of auditing: $c = 7455$ (rub.);
- unit cost of information injection: $c_{inf} = 10\%c = 745.5$ (rub);
- stopping point of the iteration process:
$$\sqrt{\sum_{i=1}^{n}(x_i(t) - x_i(t+1))} \leq 3 \cdot 10^{-2};$$
- conducting experiment: 10^2 repetitive experiments for each initial distribution.

Numerical simulation: parameters

The distribution of the income among the population of Russian Federation in 2018 [Russian Federation State Statistics Service, 2019]:

Table: Two modeled groups and average income

group	income interval (rub./month)	average income (rub.)	share of population (%)
L	less 25000	$L = 12500$	51
H	more 25000	$H = 50000$	49