PROBABILISTIC MODEL OF CULTURAL PARTICIPATION IMPACT ON SOCIAL CAPITAL
Introduction

• Although the significance of cultural processes in the society is widely recognized, there is still no clarity regarding the plausible mechanism of influence of these processes.

• Evaluation of the influence of culture on social capital is mainly based on two methodological paradigms: measuring the dynamics of cultural processes and measuring the social capital.

• This work attempts to structure a probabilistic model of social impact of cultural events, which may then be useful for determining the effects of cultural participation on social capital within a community context.
Cultural participation modelling. 
Profiles of actors in cultural participation

• The entire group of actors is modelled by a set of variable collections describing the individual actors.

• Suppose, for instance, that there are $K$ different profiles of actors available, then probability of a randomly selected actor to possess the $k^{th}$ profile is $q_k$, $1 \leq k \leq K$, here $q_1 + q_2 + \ldots + q_K = 1$. 
Cultural participation modelling. Cultural events field

Let us define a cultural events field consisting of a list of cultural events taking place in the given community, whose impact on social capital is under consideration. We will consider the cultural events field consisting of $m$ different types of events.

Denote the indicator of participation of some $k^{\text{th}}$ profile actor in the event of $i^{\text{th}}$ type by $\chi_i^k$, namely $\chi_i^k = 1$ if the participation took place, and $\chi_i^k = 0$ if it did not. Assume, for simplicity, that $\chi_i^k \cdot \chi_j^k = 0$ if $i \neq j$. Then, conditional probabilities of participation in the event make up the matrix of probabilities of preferences

$$\Pr(\chi_i^k = 1| i, k) = p_i^k,$$

where $1 \leq k \leq K, 1 \leq i \leq m$. 


Cultural participation modelling.
Modelling the perceptions of actors towards flows of cultural events (1)

• The realized sequence of events from a cultural events field presents us with the flow of cultural events that interact with community affecting the social capital of individual actors and the whole community.

• Naturally, dynamics of social impact in a population depends on the type of cultural events. In the simplest case, one can consider the stationary flow of independent events (SFIE). Such flow consists of events randomly and independently generated from the given cultural field.

• Other options include non-stationary flows of dependent cultural events and stationary and non-stationary flows of multicultural events.
Cultural participation modelling. Modelling the perceptions of actors towards flows of cultural events (2)

Stationary flow of independent events of $m$ types can be presented by event probabilities $(P_1, P_2, \ldots, P_m)$, here $P_1 + P_2 + \ldots + P_m = 1$. Hence, the probability of the $k$th profile actor to participate in the event of $i$th type following from some SFIE is expressed by means of conditional probabilities (1) in the following way:

$$\Pr(\chi_i^k = 1|k) = p_i^k = P_i \cdot P_i^k,$$

where $1 \leq k \leq K$, $1 \leq i \leq m$. Respectively, the probability of no participation of the $k$th profile actor is $\Pr(\sum_{i=1}^{m} \chi_i^k = 0|k) = 1 - \sum_{i=1}^{m} P_i \cdot P_i^k$. 
Modelling the social impact of cultural participation. Measurement of social capital (1)

OECD identifies four main ways in which the concept of “social capital” can be developed and measured:

- Personal relationships;
- Social network support;
- Civic engagement;
- Trust and cooperative norms.
Modelling the social impact of cultural participation. Measurement of social capital (2)

The analysis of research findings allows us to identify the following **basic assumptions** for creating the model of impact of cultural processes on social capital according to the OECD methodology:

1. There exists a **quantitative relationship** between participation in cultural processes and social capital.

2. The influence of cultural processes is **non-linear** and depends on **accumulated social capital**, whose indexes are limited by 0 and some maximal value. In the period of accumulation of social capital, this influence is more significant, and as social capital increases, certain “saturation” or “stabilization” can be tracked, namely, further participation in cultural processes does not lead to a significant increase in social capital.

3. The impact of a particular cultural event is **insignificant** as compared to the overall impact of the flow of cultural events, which, in its turn, is the sum of the effects of individual cultural events.

Moreover, one has to take into account that social capital is not only affected by participation in considered cultural events, but also by **other** economic, psychological, political, etc. **factors**.
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (1)

Assume that social capital is measured in accordance to the OECD methodology by the following indexes:

- Personal relationships index $C_{PR}, 0 \leq C_{PR} \leq 60$;
- Social network support index $C_{SNS}, 0 \leq C_{SNS} \leq 60$;
- Civic engagement index $C_{CE}, 0 \leq C_{CE} \leq 60$;
- Trust and cooperative norms index $C_{TCN}, 0 \leq C_{TCN} \leq 60$.

Note that the total social capital might be measured as the sum of all social capital indexes.
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (2)

Let us consider a set of $N$ actors that are influenced by a cultural events flow spread over time period $T$.

Assume, for simplicity, the time interval $T$ being a conjunction of discrete time units $\Delta t$, during which only one event from the field consisting of $m$ types of cultural events takes place.

Let each actor at some time moment be distinguished by certain value $C$ of some social capital index before the event and certain value $C_\Delta$ afterwards. Although the limiting value of social capital according to the methodology under consideration is 60, taking into account naturally existing differences between people, one can consider that this limit is individually distributed and, thus, denote it for certain actor by $A$, $0 < A \leq 60$.

Then, important information is provided by the social capital development ratio (SCDR) which describes how an actor is able to assimilate its potential:

$$D = \frac{C}{A-C},$$

where $C$ is the measured value of some actor’s social capital index, here $0 < C < A$. 
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (3)

Let us denote the fact of participation of the actor in some cultural event taking place in the community during time unit $\Delta t$ by $\chi = 1$, and no participation by $\chi = 0$.

The change of SCDR should depend on the actor’s participation in the event, thus, as the simplest case, one can consider that this ratio, having value $D$ before the event, is changed to value $D_\Delta = D + \Delta D$ after the event in the following way:

$$D_\Delta = \begin{cases} D \cdot (1 + (w_0 + \xi) \cdot \Delta t) & \text{if } \chi = 0 \\ D \cdot (1 + (b + \xi) \cdot \Delta t) & \text{if } \chi = 1 \end{cases} \quad (4)$$

where $w_0$ and $b$ are certain constants, $w_0$ describes the change of social capital if there is no participation of the agent in the event, $b$ is the expected impact of the event on social capital and $\xi$ represents the overall impact of other factors.
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (4)

Since the impact is analyzed through sufficiently large number of events, it is assumed that the impact of one separate event is small, i.e. $|w_0 \cdot \Delta t| << 1$, $|b \cdot \Delta t| << 1$.

The fact that participation in the event is expected to change the social capital positively, and no participation is acting more negatively, is modelled respectively assigning certain values: $w_0 \leq 0$, $b > 0$.

Due to the complex nature of overall factors affecting the social capital, it is reasonable to consider their entire impact $\xi$ distributed with respect to Gaussian law $N(\epsilon, \tau)$, where $\epsilon$ and $\tau$ are respective mean and variance.
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (5)

Assuming cultural events field consisting of events of \( m \) different types, and that each actor can take part only in one event at a given time moment, one can rewrite (4) as the following relationship between the change of SCDR before and after the event:

\[
D_\Delta = D \cdot (1 + (w_0 + w_1 \cdot \chi_1 + \ldots + w_m \cdot \chi_m + \xi)) \cdot \Delta t, \tag{5}
\]

where \( \chi_i \) is the indicator of participation in the event of \( i^{th} \) type, \( w_i = b_i - w_0 \), \( b_i \) is the expected impact of this event, \( 1 \leq i \leq m \).

Now, following the general definition (3), one can introduce the SCDR of social capital indexes, measured according to the OECD recommendations:

\[
D^{PR} = \frac{C^{PR}}{A^{PR} - C^{PR}}, \tag{6}
\]

\[
D^{SNS} = \frac{C^{SNS}}{A^{SNS} - C^{SNS}}, \tag{7}
\]

\[
D^{CE} = \frac{C^{CE}}{A^{CE} - C^{CE}}, \tag{8}
\]

\[
D^{TCN} = \frac{C^{TCN}}{A^{TCN} - C^{TCN}}. \tag{9}
\]
Modelling the social impact of cultural participation. 
Finite-difference model of social impact of cultural participation (6)

Proposition 1. Assume that an actor is influenced during time unit $\Delta t$ by one event only, belonging to one from $m$ types, whose particular impact is small, namely $|w_i \cdot \Delta t| << 1$, where $w_i$, $0 \leq i \leq m$, are certain weights. Then changes of SCDR due to participation in this event are given by the following expressions:

\[ D^{PR}_\Delta = D^{PR} \cdot (1 + (w^{PR}_0 + \chi_1 \cdot w^{PR}_1 + \chi_2 \cdot w^{PR}_2 + \ldots + \chi_m \cdot w^{PR}_m + \xi^{PR}) \cdot \Delta t), \]
\[ D^{SNS}_\Delta = D^{SNS} \cdot (1 + (w^{SNS}_0 + \chi_1 \cdot w^{SNS}_1 + \chi_2 \cdot w^{SNS}_2 + \ldots + \chi_m \cdot w^{SNS}_m + \xi^{SNS}) \cdot \Delta t), \]
\[ D^{CE}_\Delta = D^{CE} \cdot (1 + (w^{CE}_0 + \chi_1 \cdot w^{CE}_1 + \chi_2 \cdot w^{CE}_2 + \ldots + \chi_m \cdot w^{CE}_m + \xi^{CE}) \cdot \Delta t), \]
\[ D^{TCN}_\Delta = D^{TCN} \cdot (1 + (w^{TCN}_0 + \chi_1 \cdot w^{TCN}_1 + \chi_2 \cdot w^{TCN}_2 + \ldots + \chi_m \cdot w^{TCN}_m + \xi^{TCN}) \cdot \Delta t), \]

where indicators of participation satisfy: $\chi_i \in \{0; 1\}$, $0 \leq \chi_1 + \chi_2 + \ldots + \chi_m \leq 1$, $1 \leq i \leq m$, and $\xi = (\xi^{PR}, \xi^{SNS}, \xi^{CE}, \xi^{TCN})$ is multinormally distributed $N(\varepsilon, \tau)$ vector.
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (7)

The relationship between SCDR and social capital indexes is derived as follows:

\[ C_{PR} = A_{PR} \cdot \frac{D_{PR}}{1 + D_{PR}}, \]  \hspace{1cm} (15)

\[ C_{SNS} = A_{SNS} \cdot \frac{D_{SNS}}{1 + D_{SNS}}, \]  \hspace{1cm} (16)

\[ C_{CE} = A_{CE} \cdot \frac{D_{CE}}{1 + D_{CE}}, \]  \hspace{1cm} (17)

\[ C_{TCN} = A_{TCN} \cdot \frac{D_{TCN}}{1 + D_{TCN}}. \]  \hspace{1cm} (18)
Modelling the social impact of cultural participation. Finite-difference model of social impact of cultural participation (8)

Estimation of model parameters and model validation can be carried out by using:

- expert knowledge, when experts in the field of culture realize their opinions about respective impact of certain kinds of cultural events on the considered index of social capital;
- statistical data of measurement of participation in cultural events and measurement of social capital in community as well as surveying the participants of the events;
- analysis of unstructured internet data, such as social network exploring or qualitative content analysis methods.
Computer simulation of social impact of cultural processes: A case study (1)

• The simulation was confined by the field of cultural events composed of three types of events \( m = 3 \), namely, higher culture, popular culture and sport.

• Using statistical data, \( K = 3 \) profiles of actor preferences for participation in cultural events were established, namely those that prefer higher culture, those who take preference in popular events, and the third group, qualified as passive, whose participation and interests in culture are low.

• \( q_k, P_i, P_i^k \) and \( w_i \) were estimated using statistical data, here \( 1 \leq k \leq K, 1 \leq i \leq m \).
Computer simulation of social impact of cultural processes: A case study (2)

Table 1. Distribution of frequencies of actor profiles, probabilities of events and probabilities of preferences.

<table>
<thead>
<tr>
<th>Types of actors</th>
<th>Frequencies of actor profiles, $q_k$</th>
<th>$X_1$ (higher culture)</th>
<th>$X_2$ (pop culture)</th>
<th>$X_3$ (sport)</th>
<th>Probabilities of event types, $P_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher</td>
<td>0.35</td>
<td>0.05</td>
<td>0.72</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Popular</td>
<td>0.39</td>
<td>0.67</td>
<td>0.57</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Passive</td>
<td>0.26</td>
<td>0.12</td>
<td>0.73</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Weights of impacts of participation in cultural events

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{P1}^{PR}$</td>
<td>-0.032</td>
<td>Reduction coefficient of $D^{PR}$</td>
</tr>
<tr>
<td>$w_{P2}^{PR}$</td>
<td>0.2</td>
<td>Sensitivity of $D^{PR}$ to participation in higher culture event $X_1$</td>
</tr>
<tr>
<td>$w_{P3}^{PR}$</td>
<td>0.26</td>
<td>Sensitivity of $D^{PR}$ to participation in pop culture event $X_2$</td>
</tr>
<tr>
<td>$w_{P3}^{SNS}$</td>
<td>0.18</td>
<td>Sensitivity of $D^{SNS}$ to participation in sport event $X_3$</td>
</tr>
<tr>
<td>$w_{SNS1}^{SNS}$</td>
<td>-0.009</td>
<td>Reduction coefficient of $D^{SNS}$</td>
</tr>
<tr>
<td>$w_{SNS2}^{SNS}$</td>
<td>0.039</td>
<td>Sensitivity of $D^{SNS}$ to participation in higher culture event $X_1$</td>
</tr>
<tr>
<td>$w_{SNS3}^{SNS}$</td>
<td>0.11</td>
<td>Sensitivity of $D^{SNS}$ to participation in pop culture event $X_2$</td>
</tr>
<tr>
<td>$w_{SNS4}^{SNS}$</td>
<td>0.047</td>
<td>Sensitivity of $D^{SNS}$ to participation in sport event $X_3$</td>
</tr>
<tr>
<td>$w_{CF0}^{CF}$</td>
<td>-0.003</td>
<td>Reduction coefficient of $D^{CF}$</td>
</tr>
<tr>
<td>$w_{CF1}^{CF}$</td>
<td>0.014</td>
<td>Sensitivity of $D^{CF}$ to participation in higher culture event $X_1$</td>
</tr>
<tr>
<td>$w_{CF2}^{CF}$</td>
<td>0.012</td>
<td>Sensitivity of $D^{CF}$ to participation in pop culture event $X_2$</td>
</tr>
<tr>
<td>$w_{CF3}^{CF}$</td>
<td>0.039</td>
<td>Sensitivity of $D^{CF}$ to participation in sport event $X_3$</td>
</tr>
<tr>
<td>$w_{CF4}^{CF}$</td>
<td>-0.024</td>
<td>Reduction coefficient of $D^{CF}$</td>
</tr>
<tr>
<td>$w_{CF1}^{CF}$</td>
<td>0.15</td>
<td>Sensitivity of $D^{CF}$ to participation in higher culture event $X_1$</td>
</tr>
<tr>
<td>$w_{CF2}^{CF}$</td>
<td>0.18</td>
<td>Sensitivity of $D^{CF}$ to participation in pop culture event $X_2$</td>
</tr>
<tr>
<td>$w_{CF3}^{CF}$</td>
<td>0.14</td>
<td>Sensitivity of $D^{CF}$ to participation in sport event $X_3$</td>
</tr>
</tbody>
</table>
Computer simulation of social impact of cultural processes: A case study (3)

• In total, 100 scenarios of cultural event flows influencing the population consisting of \( N = 200 \) actors during time period \( T = 100 \) with time unit \( \Delta t = 1 \) were simulated.

• The initial values of SCDR were taken as \( D_0^{PR} = 1, D_0^{SNS} = 1, D_0^{CE} = 1, D_0^{TCP} = 1 \).

• The potential social capital values \( A \) of individuals were simulated according to \( \beta \)-distribution with parameters \( \alpha = 5, \beta = 1 \).

• The impact of environment, modelled as normally distributed, was taken with parameters \( \varepsilon = (0, 0, 0, 0), \tau = (0.01, 0.01, 0.01, 0.01) \).
Computer simulation of social impact of cultural processes: A case study (4)

Dynamic dependencies of social capital
Computer simulation of social impact of cultural processes: A case study (5)

Dynamic dependencies of social capital
Computer simulation of social impact of cultural processes: A case study (6)
Computer simulation of social impact of cultural processes: A case study (7)
Computer simulation of social impact of cultural processes: A case study (8)

Table 3. Sensitivity to weights of impact

<table>
<thead>
<tr>
<th>Change in w values</th>
<th>Change in average social capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.91%</td>
</tr>
<tr>
<td>20%</td>
<td>1.92%</td>
</tr>
<tr>
<td>50%</td>
<td>3.44%</td>
</tr>
<tr>
<td>-10%</td>
<td>-0.83%</td>
</tr>
<tr>
<td>-20%</td>
<td>-2.34%</td>
</tr>
<tr>
<td>-50%</td>
<td>-8.58%</td>
</tr>
</tbody>
</table>
Concluding words

• A probabilistic model relating cultural participation with social capital was constructed using a few quite simple, yet highly probable assumptions.

• The model was tested on the mixture of statistical data and expert knowledge. The presented case showed that the model acts as initially intended.

• Such model can be used on its own (as is or extended) for the analysis of various real-life cases or implemented in, for example, agent-based models.

• Further validation should be carried out to find out if the current set of assumptions describes the reality well enough.
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